TIME AND DISTANCE

The **Speed** of a body is the rate at which it is moving that is distance travelled in unit time. It is a measure by the distance, a moving body would cover in a given time. Thus, we see that the distance covered by a moving body depends on the speed of the body or person and the time taken.

Basic Rules :

(*i*) More distance, more time; at the same speed.
(*ii*) More speed; less time; for the same distance.
(*iii*) More speed; more distance in the same time.
(*iv*) If the speed of a body is changed in the ratio $a : b$, the ratio of time taken to cover a given distance changes in the ratio $b : a$.

Formulae : (*i*) Distance = Speed × Time

(*ii*) $\text{Speed} = \dfrac{\text{Distance}}{\text{Time}}$

and, (*iii*) $\text{Time} = \dfrac{\text{Distance}}{\text{Speed}}$

Conversion of Units :

$$x \text{ km/hr} = x\frac{1000}{3600}\text{ m/s}\left(\begin{array}{l}\because 1\text{ km} = 1000\text{ m}\\ \ \ 1\text{ hr} = 3600\text{ seconds}\end{array}\right)$$

or, $\mathbf{x}$ **km/hr** $= \dfrac{5}{18}x$ **m/s**

and, $\mathbf{x}$ **m/s** $= \dfrac{18}{5}x$ **km/hr**

[Here,
km = Kilometre; m = Metre; hr = Hour; S = Second]

Average Speed : If a certain distance is covered in parts at different speeds, the average speed is given by,

$$\text{Average Speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$

Velocity : The speed of a moving body is called its velocity if the direction of motion is also taken into consideration. Though, speed and velocity are interchangeably used in daily life, the two are different quantities. It is given by

$$\text{Velocity} = \frac{\text{Net displacement of the body}}{\text{Time taken}}.$$

Relative Speed :

(*a*) ***Bodies moving in the same direction :*** (*i*) If two bodies move in the **same direction,** the relative speed of one with respect to the other is the difference of their speeds. For example, if the two cars A and B move in the same direction at speeds of 40 km/hr and 30 km/hr respectively, the relative speed of A with respect to B is (40 – 30) = 10 km/hr.

(*ii*) When two bodies move in the same direction, the distance between them increases/decreases at the rate of difference in their speeds. In other words, increase (or decrease) in distance between them after time t is equal to the product of difference in their speeds and time t.

Ex. 1. Two cars A and B start from the same point at speeds 40 km/hr and 30 km per hr respectively in the same direction. Find the distance between them after 3 hours.

Sol. Difference in speeds or Relative speed
= 40 – 30 = 10 km/hr.
∴ Distance between A and B after 3 hours
= 10 × 3 = 30 kms **Ans.**

(*b*) ***Bodies moving in the opposite directions :*** (*i*) Relative speed of one with respect to the other is sum of their speeds.

(*ii*) Increase or decrease in distance between them equals product of their relative speed and time.

(*iii*) The distance between two bodies moving towards each other will get reduced at the rate of their relative speed (i.e., sum of their speeds). The time of their meeting (or crossing) is given by,

$$\text{Meeting time} = \frac{\text{Initial distance between the two bodies}}{\text{Sum of their speeds}}.$$

Ex. 2. A person covers 800 metres in 160 seconds. Find his speed.

Sol. $\text{Speed} = \dfrac{\text{Distance}}{\text{Time}} = \dfrac{800\text{ m}}{160\text{ s}} = 5\text{ m/s}$ **Ans.**

Ex. 3. A person runs at 5 km/hr. How much distance he will cover in $4\frac{1}{2}$ hours?

Sol. Time $= 4\frac{1}{2}$ hrs $= \frac{9}{2}$ hrs

Distance covered = Speed × Time

$= 5 \times \frac{9}{2} = \frac{45}{2} = 22\frac{1}{2}$ kms **Ans.**

Ex. 4. A person walks at 3 km/hr. In how much time he will cover 900 m?

Sol. : $3 \text{ km/hr} = 3 \times \frac{5}{18}$ m/s $= \frac{5}{6}$ m/s

$\text{Time} = \frac{900 \text{ m}}{\frac{5}{6} \text{ m/s}} = 1080$ seconds. = 18 minutes **Ans.**

Note : During all calculations the respective units for distance, time and speed must be made consistent i.e, all distances must be expressed in the same unit, either metres or kilometres or yards as the case may be. Similarly, all time values must be expressed in the same unit, either hours, minutes or seconds.

Ex. 5. A person covers a distance d_1 kms at s_1 km/hr and then d_2 kms at s_2 km per hr. Find his average speed during the whole journey.

Sol. Time taken to travel d_1 kms at s_1 km/hr : t_1 $= \frac{d_1}{s_1}$ hr.

Time taken to travel d_2 kms at s_2 km/hr : t_2 $= \frac{d_2}{s_2}$ hr.

Total time taken $= t_1 + t_2$

$= \left(\frac{d_1}{s_1} + \frac{d_2}{s_2}\right)$ hr. $= \left(\frac{s_2 d_1 + s_1 d_2}{s_1 s_2}\right)$ hr.

Total distance covered $= (d_1 + d_2)$ kms.

Therefore, Average speed $= \frac{\text{Total distance covered}}{\text{Total time taken}}$

$= \frac{d_1 + d_2}{(s_1 d_1 + s_1 d_2)/s_1 s_2}$ hr

or, Average speed $= \frac{s_1 s_2 (d_1 + d_2)}{s_2 d_1 + s_1 d_2}$ km/hr

Special Case : In the case of return journey or other similar cases where $d_1 = d_2 = d$ (say), then

$$\text{Average speed} = \frac{s_1 s_2 (d + d)}{s_2 d + s_1 d} = \frac{2d\, s_1 s_2}{d\,(s_1 + s_2)}$$

$$\text{or, Average speed} = \frac{2\, s_1 s_2}{s_1 + s_2} = \frac{2 \times \text{Product of speeds}}{\text{Sum of speeds}}$$

$$= \frac{\text{Product of speeds}}{\text{Average of speeds}}$$

Note : In such cases where $d_1 = d_2$, the average speed for the whole journey is independent of the distance.

Ex. 6. A person leaves his house in the morning for office and returns to his house in the evening. In the morning he travels at a speed of 35 kms/hr but during return in the evening his speed is only 25 kms/hr. Find his average speed for the whole journey.

Sol. Here the two distances covered at two different speeds are equal i.e., $d_1 = d_2$.

Therefore, Average speed $= \frac{2 \times 35 \times 25}{35 + 25}$

$= \frac{2 \times 35 \times 25}{60} = \frac{175}{6}$ km/hr

or $= 29\frac{1}{6}$ km/hr **Ans.**

Note : (*i*) If a person covers $\frac{1}{n}$th part of the total journey at speed S_1 and the remaining journey at speed S_2, then his average speed for the total journey is given by,

$$\boxed{\text{Average Speed} = \frac{n S_1 S_2}{(n-1) S_1 + S_2}}$$

(*ii*) If a person covers $\frac{m}{n}$th part of the total journey at speed S_1 and the remaining journey at speed S_2, then his average speed for the total journey is given by,

$$\boxed{\text{Average Speed} = \frac{n S_1 S_2}{(n-m) S_1 + m S_2}}$$

In both the above cases the total journey distance does not appear in the final expression.

If different parts travelled at different speeds are expressed as fractions of the total distance, for example, *m*th part at speed S_1, *n*th part at S_2, *p*th part at S_3 and so on, then

$$\boxed{\text{Average speed} = \frac{1}{\frac{m}{S_1} + \frac{n}{S_2} + \frac{p}{S_3} + \dots\dots\dots}}$$

However, care should be taken to include all the parts of the total distance in the denominator.

Ex. 7. A boy goes to school from his house at speed S_1 km/hr and is late by l_1 hour. If he goes at S_2 km/hr he reaches l_2 hours early. Show that the distance between his house and school is $\left(\frac{S_1 S_2}{S_2 - S_1}\right)(l_1 + l_2)$ **km.**

Sol. Let the distance between house and school be D kms.

Time taken to reach school at speed $S_1 = \frac{D}{S_1}$

and time taken to reach schoool at speed $S_2 = \frac{D}{S_2}$

We can write,

$$\frac{D}{S_1} - l_1 = \frac{D}{S_2} + l_2$$

$$\text{or, } D\left[\frac{1}{S_1} - \frac{1}{S_2}\right] = l_1 + l_2 \text{ or, } D\left[\frac{S_2 - S_1}{S_1 S_2}\right] = l_1 + l_2$$

$$\text{or, } D = \left(\frac{S_1 S_2}{S_2 - S_1}\right)(l_1 + l_2)$$

Note : It is appropriate to remind once again that in solving the problems the consistency of units always must be kept in mind, i.e., all time values must be expressed in the same unit, and the same for distance and speed too.

We can generalize this relation as

$$D = \frac{S_1 S_2}{S_2 - S_1}(l_1 - l_2)$$

with following **sign convention:**

(*i*) Late time : +ve ; Early time : –ve

(*ii*) If D comes –ve, ignore negative sign as distance can never be negative.

Ex. 8. Walking at $\frac{4}{5}$th of his usual speed, a person is 2 hours late. How much time he usually takes to travel the same distance?

Sol. Since the person walks at $\frac{4}{5}$th of his usual speed, the time he takes is $\frac{5}{4}$th of his usual time.

$\left[\text{Time} \propto \frac{1}{\text{speed}} \text{ when distance covered remains constant.}\right]$

Let the usual time taken be t hours.

Then we can write $\frac{5}{4}t = t + 2$

or, $\frac{t}{4} = 2$

or, $t = 8$ hrs **Ans.**

Ex. 9. A person covers his onward journey at speed S_1 km/hr and the return journey of equal distance at speed S_2 km/hr. If the total time taken during the to and fro journey is T hours, what is the one way journey distance?

Sol. Let the one way journey distance be D kms.

Time taken during onward journey = $t_1 = \frac{D}{S_1}$ hrs.

Time taken during return journey = $t_2 = \frac{D}{S_2}$ hrs.

Total time taken during the complete to and fro journey :

$$T = t_1 + t_2 = \frac{D}{S_1} + \frac{D}{S_2} = \frac{DS_2 + DS_1}{S_1 S_2} = \frac{D(S_2 + S_1)}{S_1 S_2}$$

$$\text{or, } T = D\left(\frac{S_2 + S_1}{S_1 S_2}\right) \text{ or, } D = T\left(\frac{S_1 S_2}{S_2 + S_1}\right)$$

∴ One way Journey Distance

$$= \text{Total time taken} \times \frac{\text{Product of two speeds}}{\text{Sum of two speeds}}$$

Note : This expression appears to be similar to that for average speed. In fact, we can derive this expression from the relation for average speed derived in Ex. 12.

$$\text{Average speed } = \frac{2S_1 S_2}{S_1 + S_2}$$

$$\text{But, Average speed given} = \frac{2D}{T}$$

$$\therefore \frac{2D}{T} = \frac{2S_1 S_2}{S_1 + S_2}$$

$$\text{or, } D = T\left(\frac{S_1 S_2}{S_1 + S_2}\right)$$

Ex. 10. A person walks from his house to his office at 4 km/hr and returns to his house at 2 km/hr. If he spends total 6 hours on his to and fro travelling, what is the distance between his house and office?

Sol. $S_1 = 4$ km / hr, $S_2 = 2$ km/hr, $T = 6$ hours.

$$D = T\left(\frac{S_1 S_2}{S_1 + S_2}\right) = 6 \times \frac{4 \times 2}{4 + 2} = 8 \text{ kms } \textbf{Ans.}$$

Ex. 11. Car A starts from P at time t_1 and reaches Q at t_2. Another car B starts from Q at t_3 (later than t_1) and reaches P at t_4. At what time do the two cars meet on their way?

Sol. Let the distance PQ be D.

$$A\text{'s speed} = \frac{D}{t_2 - t_1}$$

$$B\text{'s speed} = \frac{D}{t_4 - t_3}$$

At the time of start of B, the distance already covered by $A = (t_3 - t_1) \times \frac{D}{t_2 - t_1}$

$$\text{Remaining distance} = D - \left(\frac{t_3 - t_1}{t_2 - t_1}\right)D = \frac{t_2 - t_3}{t_2 - t_1}D$$

Relative speed of approach of two cars

$$= \frac{D}{t_2 - t_1} + \frac{D}{t_4 - t_3} = \frac{(t_4 - t_3) + (t_2 - t_1)}{(t_2 - t_1)(t_4 - t_3)} \times D$$

Time taken by two cars to cover the remaining distance

$$= \frac{\frac{t_2 - t_3}{t_2 - t_1} D}{\frac{(t_2 - t_1) + (t_4 - t_3)}{(t_2 - t_1)(t_4 - t_3)} D} = \frac{(t_2 - t_3)(t_4 - t_3)}{(t_2 - t_1) + (t_4 - t_3)}$$

The time at which two cars will meet

$$= t_3 + \frac{(t_2 - t_3)(t_4 - t_3)}{(t_2 - t_1) + (t_4 - t_3)}$$

or, They will meet at

$$= \text{Second's starting time} + \frac{(\text{First's arrival time} - \text{Second's starting time})(\text{Time taken by second})}{\text{Sum of time taken by both}}$$

Ex. 12. A person covers a certain distance in 6 hours if he travels at 40 km/hr. At what speed he must travel if he has to cover the same distance in 4 hours?

Sol. $D = S \times T$

Since, D remains constant in the two cases we can write,

$D = S_1 T_1 = S_2 T_2$ or, $S_2 = S_1 \frac{T_1}{T_2}$

$\therefore S_2 = 40 \times \frac{6}{4}$ or, $S_2 = 60$ km/hr. **Ans.**

(Speed and time are inversely proportional if distance covered remains constant. More speed, less time and *vice-versa*).

EXERCISE

1. Buses start from a bus terminal with a speed of 20 km/hr at intervals of 10 minutes. What is the speed of a man coming from the opposite direction towards the bus terminal if he meets the buses at intervals of 8 minutes ?
(1) 3 km/hr (2) 4 km/hr
(3) 5 km/hr (4) 7 km/hr

2. By walking at $\frac{3}{4}$ of his usual speed, a man reaches his office 20 minutes later than his usual time. The usual time taken by him to reach his office is
(1) 75 minutes (2) 60 minutes
(3) 40 minutes (4) 30 minutes
(SSC Graduate Level Tier-I Exam, 16.05.2010)

3. A and B started at the same time from the same place for a certain destination. B walking at $\frac{5}{6}$ of A's speed reached the destination 1 hour 15 minutes after A. B reached the destination in
(1) 6 hours 45 minutes (2) 7 hours 15 minutes
(3) 7 hours 30 minutes (4) 8 hours 15 minutes

4. Two men start together from the some place in the same direction to go round a circular path. If one takes 10 minutes and the other takes 15 minutes to make one complete round they will meet after
(1) 30 minutes (2) 33 minutes
(3) 40 minutes (4) 45 minutes

5. A man takes 6 hours 15 minutes in walking a distance and riding back to the starting place. He could walk both ways in 7 hours 45 minutes. The time taken by him to ride both ways, is
(1) 4 hours (2) 4 hours 30 minutes
(3) 4 hours 45 minutes (4) 5 hours
(SSC Graduate Level Prelim Exam, 27.07.2008)

6. A man completed a certain journey by a car. If he covered 30% of the distance at the speed of 20km/hr, 60% of the distance at 40km/hr and the remaining distance at 10km/hr; his average speed for the whole journey was
(1) 25 km/hr (2) 28 km/hr
(3) 30 km/hr (4) 33 km/hr

7. A boy has a few coins of denominations 50 paise, 25 paise and 10 paise in the ratio 1 : 2 : 3. If the total amount of the coins is Rs 6.50, the number of 10 paise coins is
(1) 5 (2) 10
(3) 15 (4) 20

8. If p men working p hours per day for p days produce p units of work, then the units of work produced by n men working n hours a day for n days is
(1) $\frac{p^2}{n^2}$ (2) $\frac{p^3}{n^2}$
(3) $\frac{n^2}{p^2}$ (4) $\frac{n^3}{p^2}$

9. From two places, 60 km apart, A and B start towards each other at the same time and meet each other after 6 hours. Had A travelled with $\frac{2}{3}$ of his speed and B travelled with double of his speed, they would have met after 5 hours. The speed of A is
(1) 4 km/hr. (2) 6 km/hr.
(3) 10 km/hr. (4) 12 km/hr.

10. A, B, and C start together from the same place to walk round a circular path of length 12km. A walks at the rate of 4 km/hr., B 3 km./hr and C $\frac{3}{2}$ km/hr. They will meet together at the starting place at the end of
(1) 10 hours (2) 12 hours
(3) 15 hours (4) 24 hours

ANSWERS

1. (3)	**2.** (2)	**3.** (3)	**4.** (1)	**5.** (3)
6. (1)	**7.** (3)	**8.** (4)	**9.** (2)	**10.** (4)